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► Soal Kuis 3 KPB

Diberikan integral berikut : $\int_0^1 \int_{x/2}^{1-x/2} \int_0^{2-x-2y} \rho \, dz \, dy \, dx$, dengan $\rho = k\ell$

d. Gambar daerah Integrasinya.

→ Batas pada Sumbu-z

• Batas atas : $z = 2 - x - 2y$

• Batas bawah : $z = 0$

→ Batas pada sumbu-x

• $0 \leq x \leq 1$

→ Batas pada Sumbu-y

• Batas atas : $y = 1 - \frac{x}{2}$

• Batas bawah : $y = \frac{x}{2}$

* Untuk menggambar, gambar mulai dimensi 3

• $z = 2 - x - 2y$

Saat $z = 0 \rightarrow x = 2 - 2y$

• $x = 0 \rightarrow y = 1$

• $y = 0 \rightarrow x = 2$

Saat $y = 0 \rightarrow z = 2 - x$

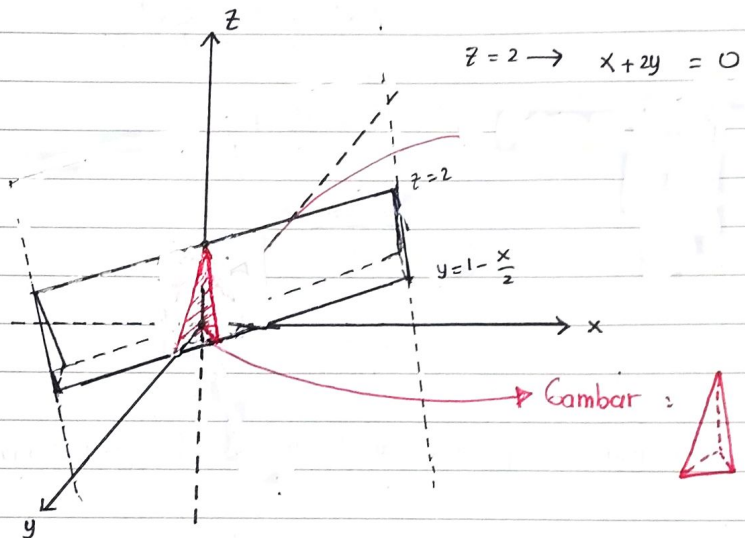
• $z = 0 \rightarrow x = 2$

• $x = 0 \rightarrow z = 2$

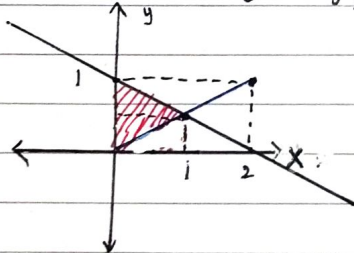
Saat $x = 0 \rightarrow z = 2 - 2y$

• $z = 0 \rightarrow y = 1$

• $y = 0 \rightarrow z = 2$



► Dalam dimensi 2 (x dan y)



$$y = 1 - \frac{x}{2}$$

$$x = 0 \rightarrow y = 1$$

$$y = 0 \rightarrow x = 2$$

$$y = \frac{x}{2} \rightarrow x = 0, \text{ maka } y = 0$$

$$x = 1 \rightarrow y = \frac{1}{2}$$

$$y = 1 \rightarrow x = 2$$

b. Dapatkan Nilai Integrasinya.

$$\begin{aligned} \int_0^1 \int_{x/2}^{1-x/2} \int_0^{2-x-2y} \rho \, dz \, dy \, dx &= \int_0^1 \int_{x/2}^{1-x/2} \left[\int_0^{2-x-2y} kx \, dz \right] dy \, dx \\ &= \int_0^1 \int_{x/2}^{1-x/2} \left[kxz \right]_0^{2-x-2y} dy \, dx \\ &= \int_0^1 \left[\int_{x/2}^{1-x/2} kx(2-x-2y) dy \right] dx \\ &= \int_0^1 \left[\int_{x/2}^{1-x/2} (2kx - kx^2 - 2kxy) dy \right] dx \end{aligned}$$

$$= \int_0^1 \left[\int_{\frac{x}{2}}^{1-\frac{x}{2}} (2kx - kx^2) - 2kxy \, dy \right] dx$$

$$= \int_0^1 \left[(2kx - kx^2)y - kxy^2 \right]_{\frac{x}{2}}^{1-\frac{x}{2}} dx$$

$$= \int_0^1 \left[[2kx - kx^2] \left(1 - \frac{x}{2}\right) - kx \left(1 - \frac{x}{2}\right)^2 - \left[2kx - kx^2\right] \left(\frac{x}{2}\right) + kx \left(\frac{x}{2}\right)^2 \right] dx$$

$$= \int_0^1 \left[2kx - kx^2 - kx^2 + \frac{kx^3}{2} - kx + \frac{kx^2}{2} - \frac{kx^3}{4} - kx^2 + \frac{kx^3}{2} + \frac{kx^3}{4} \right] dx$$

$$= \int_0^1 kx - 2kx^2 + \frac{kx^3}{2} dx$$

$$= \left[\frac{k}{2} x^2 - \frac{2kx^3}{3} + \frac{k}{4} x^4 \right]_0^1$$

$$= \frac{k}{2} - \frac{2k}{3} + \frac{k}{4} = 0$$

$$= \frac{6k - 8k + 3k}{12}$$

$$= \frac{k}{12} //$$

C. Jelaskanlah interpretasi dari integral tersebut jika ρ adalah density benda!

► Soal Buku Bu Nur Asiyah

11. a. $\int_{x=1}^2 \int_{y=x}^{y=2x} \int_{z=0}^{z=y-x} dx dy dz$

$$\begin{aligned}
 &= \int_1^2 \int_x^{2x} \left[\int_0^{y-x} dz \right] dy dx = \int_1^2 \int_x^{2x} z \Big|_0^{y-x} dy dx \\
 &= \int_1^2 \left[\int_x^{2x} (y-x) dy \right] dx \\
 &= \int_1^2 \left[\frac{1}{2} y^2 - xy \Big|_x^{2x} \right] dx \\
 &= \int_1^2 \left[\frac{4x^2}{2} - 2x^2 - \frac{x^2}{2} + x^2 \right] dx \\
 &= \int_1^2 \left[\frac{3}{2} x^2 - x^2 \right] dx \\
 &= \int_1^2 \frac{1}{2} x^2 dx \\
 &= \frac{1}{6} x^3 \Big|_1^2 \\
 &= \frac{8}{6} - \frac{1}{6} \\
 &= \frac{7}{6} //
 \end{aligned}$$

Jawab = $\frac{7}{6}$ //

b. $\int_{z=0}^2 \int_{x=z}^2 \int_{y=8x}^{y=z} dx dy dz$

$$\begin{aligned}
 &= \int_0^2 \int_z^2 \left[\int_{8x}^z dy \right] dx dz \\
 &= \int_0^2 \int_z^2 \left[y \Big|_{8x}^z \right] dx dz \\
 &= \int_0^2 \left[\int_z^2 (z - 8x) dx \right] dz \\
 &= \int_0^2 \left[zx - 4x^2 \Big|_z^2 \right] dz \\
 &= \int_0^2 \left[(2z - 4 \cdot 4) - (z^2 - 4z^2) \right] dz \\
 &= \int_0^2 \left[2z - 16 + 3z^2 \right] dz \\
 &= \left[z^2 + z^3 - 16z \right]_0^2
 \end{aligned}$$

Jawab = -20 //

= 8 + 4 - 32 = 0

= -20 //

$$\begin{aligned}
 c). & \int_{z=1}^2 \int_{y=-1}^3 \int_{x=y+z}^{2y+z} 6y \, dx \, dy \, dz \\
 &= \int_{z=1}^2 \int_{y=-1}^3 \left[6y x \right]_{x=y+z}^{2y+z} dy \, dz \\
 &= \int_{z=1}^2 \int_{y=-1}^3 \left[6xy \right]_{y+z}^{2y+z} dy \, dz \\
 &= \int_{z=1}^2 \int_{y=-1}^3 \left[6y(2y+z) - 6y(y+z) \right] dy \, dz \\
 &= \int_{z=1}^2 \int_{y=-1}^3 \left[12y^2 + 6yz - 6y^2 - 6yz \right] dy \, dz \\
 &= \int_{z=1}^2 \left(\int_{y=-1}^3 6y^2 dy \right) dz \\
 &= \int_{z=1}^2 \left[2y^3 \right]_{y=-1}^3 dz \\
 &= \int_1^2 \left[3^3 - (-1)^3 \right] dz \\
 &= \int_1^2 54 + 2 \, dz \\
 &= 56z \Big|_1^2 \\
 &= 56(2-1) \\
 &= 56
 \end{aligned}$$

Jawab = 56 //

$$\begin{aligned}
 d). & \int_{x=1}^2 \int_{z=x}^{2x} \int_{y=0}^{\frac{1}{z}} z \, dx \, dy \, dz \\
 &= \int_{x=1}^2 \int_{z=x}^{2x} \left[\int_{y=0}^{\frac{1}{z}} z \, dy \right] dz \, dx \\
 &= \int_{x=1}^2 \int_{z=x}^{2x} \left[zy \right]_0^{\frac{1}{z}} dz \, dx \\
 &= \int_{x=1}^2 \left(\int_{z=x}^{2x} dz \right) dx \\
 &= \int_{x=1}^2 \left[z \right]_x^{2x} dx \\
 &= \int_{x=1}^2 2x - x \, dx \\
 &= \int_{x=1}^2 x \, dx \\
 &= \left[\frac{1}{2} x^2 \right]_1^2
 \end{aligned}$$

$$= \frac{1}{2} ((2)^2 - (1)^2)$$

$$= \frac{1}{2} (4 - 1)$$

$$= \frac{3}{2} //$$

Jawab = $\frac{3}{2}$ //